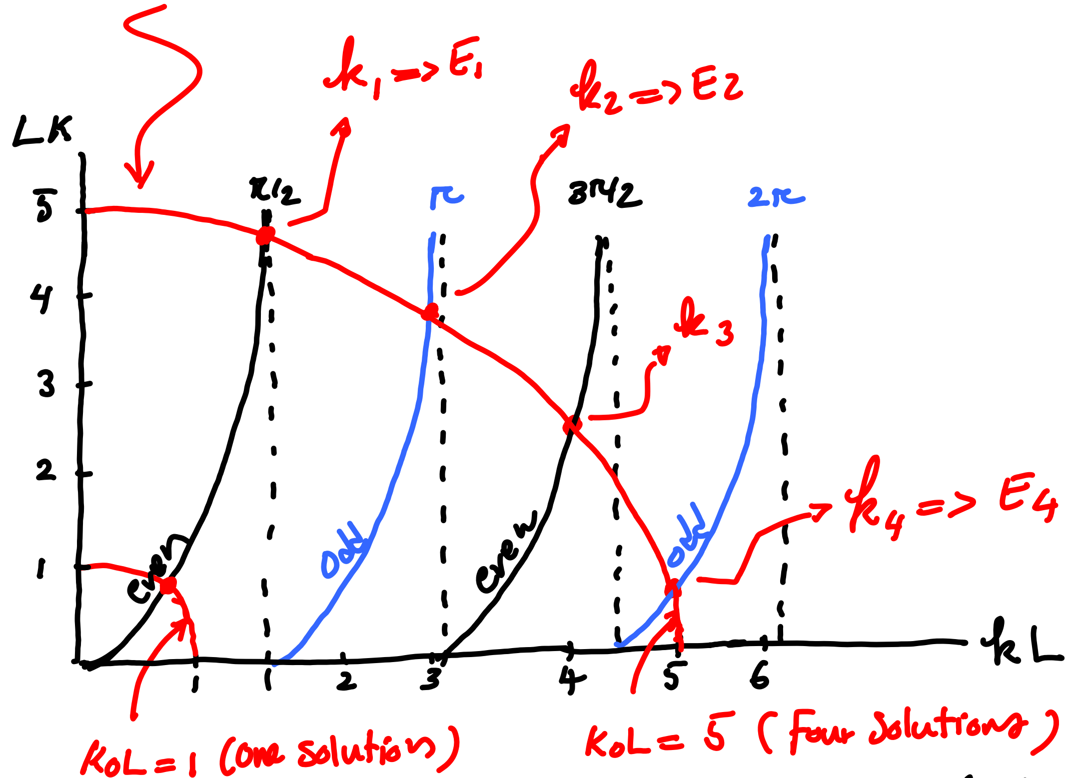
(I) $\begin{cases} kL \tan kL = KL & \text{even parity} \\ (kL)^2 + (KL)^2 = (K_0L)^2 \end{cases}$

(II) $\begin{cases} kL \cot kL = -KL & \text{odd parity} \\ (kL)^2 + (KL)^2 = (K_0L)^2 \end{cases}$



remembe: K_0L is related to the height of potential barrier:

$$K_0L = \frac{\sqrt{2mV_0}}{\hbar} L$$

So depending on V_0, there different energy levels corresponding to values of (k, K): $E = \frac{\hbar^2 k^2}{2m}$

(or: $V_0 - E = \frac{\hbar^2 K^2}{2m}$ gives same E)

<u>Let's plot the wave functions for $K_0L = 5$</u>

$K_0L = 5$ Corresponds to $K_0L = \frac{\sqrt{2mV_0}}{\hbar} L = 5 \Rightarrow V_0 = \frac{25\hbar^2}{2mL^2}$

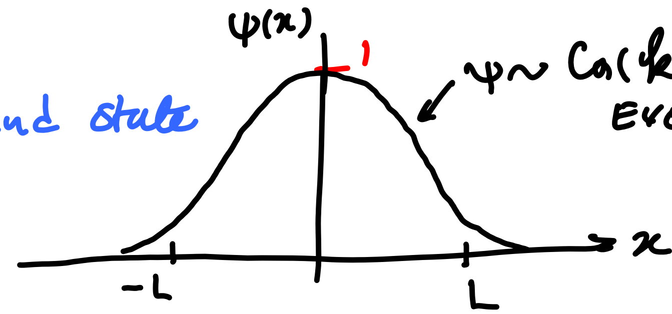
For a superlattice with $L = 5\,nm$ and effective mass $m = 0.07 m_e = 0.07 \times 9.1 \times 10^{-31}$ kg $\Rightarrow$

$V_0 = 500$ meV (typicall SL potential barriers)

Numerically solve for equs I & II $\Rightarrow$

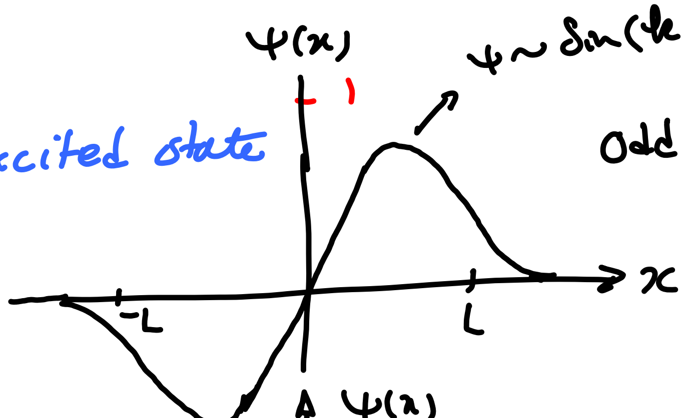
$E = 36.7,\ 144,\ 314,\ 408$ meV

Ground state



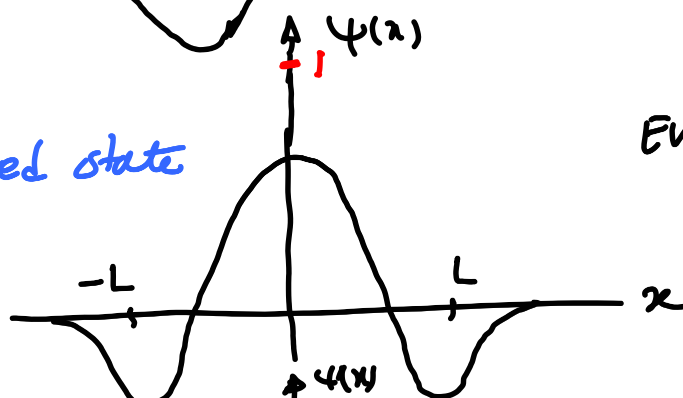
$\psi \sim \cos(kx)$
Even parity

1^{st} excited state



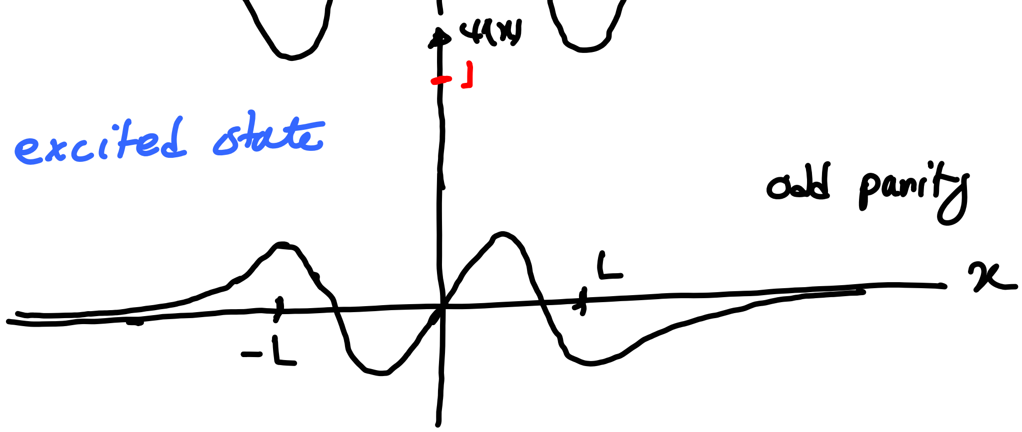
$\psi \sim \sin(kx)$
Odd parity

2^{nd} excited state



Even parity

3^{rd} excited state



odd parity

Check the limiting case

Notice that as $V_0 \longrightarrow \infty$ the intersection of the circle $(k_0 L)^2 = (kL)^2 + (\kappa L)^2$ with tan & cot curves happen at $kL = \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \dots = n\frac{\pi}{2}$ $\Rightarrow$

$$E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m} \frac{n^2 \pi^2}{4L^2}$$ which is similar to what we had for infinite potential well.